

An Explanation on Negative Mass-Square of Neutrinos

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It has been known for many years that the measured mass-square of neutrino exhibits significantly negative value. It was proposed three decades ago that neutrinos might be superluminal fermions. In this paper, we have further investigated this hypothesis. A new Dirac-type equation is established and its validity is proved, which can be used to solve the puzzle of negative mass-square of neutrinos. This equation is equivalent to two Weyl equations coupled together via nonzero mass while respecting the maximum parity violation, and it reduces to one Weyl equation when the neutrino mass is zero.

PACS number: 14.60.Lm, 14.60.Pq, 14.60.St

The square of the neutrino mass is measured in tritium beta decay experiments by fitting the shape of the beta spectrum near endpoint. In many experiments, it has been found to be significantly negative. Most recent data are listed in "Review of Particle Physics, 2000" [1] and references therein. The weighted average from two experiments reported in 1999 is

$$m^2(\nu_e) = -2.5 \pm 3.3 eV^2 \quad (1)$$

However, other nine measurements from different experiments in 1991-1995 are not used for averages. For instance, a value of $m^2(\nu_e) = -130 \pm 20 eV^2$ with 95% confidence level was measured in LLNL in 1995[1].

The negative value of the electron neutrino mass-square simply means:

$$E^2/c^2 - p^2 = m^2(\nu_e)c^2 < 0 \quad (2)$$

The right-hand side in Eq. (2) can be rewritten as $(-m_s^2c^2)$, then m_s has a positive value. Eq. (2) strongly suggests that electron neutrinos might be particles faster than light [2,3]. Furthermore, the pion decay experiment also obtained a negative value for μ -neutrinos [1].

$$m^2(\nu_\mu) = -0.016 \pm 0.023 MeV^2 \quad (3)$$

Recently, τ -neutrinos has been discovered experimentally in Fermilab, no data of $m^2(\nu_\tau)$ has yet been reported.

Based on special relativity and known as re-interpretation rule, superluminal particles were proposed by Bilaniuk et al [2-4]. The sign of 4-D world line element, ds^2 , is associated with three classes of particles. For simplicity, let $dy = dz = 0$, then

$$> 0 \quad \text{Class I (subluminal particles)}$$

$$ds^2 = c^2dt^2 - dx^2 = 0 \quad \text{Class II (photon)} \quad (4)$$

$$< 0 \quad \text{Class III (superluminal particles)}$$

For Class III particles, i.e. superluminal particle, the relation of momentum and energy is shown in Eq. (2). Notice the negative value on the right-hand side of Eq. (2) for superluminal particles. It means that p^2 is greater than $(E/c)^2$. The velocity of a superluminal particle, u_s , is greater than speed of light. The momentum and energy in terms of u_s are as follows:

$$\begin{aligned} p &= m_s u_s (u_s^2/c^2 - 1)^{-1/2} \\ E &= m_s c^2 (u_s^2/c^2 - 1)^{-1/2} \end{aligned} \quad (5)$$

where the subscript s means superluminal particle. From Eq. (5), it is easily seen that when u_s is increased, both of p and E would be decreased. This property is opposite to Class I particle.

Any physical reference system is built by Class I particles (atoms, molecules etc.), which requires that any reference frame must move slower than light. On the other hand, once a superluminal particle is created in an interaction, its speed is always greater than the speed of light. Neutrino is the most possible candidate for a superluminal particle because it has left-handed spin in any reference frame. On the other hand, anti-neutrino always has right-handed spin.

The first step in this direction is usually to introduce an imaginary mass. For instance, Chodos et al. [5] have examined the possibility that neutrino is superluminal fermion with an imaginary mass. A form of the lagrangian density for superluminal neutrinos was proposed, but they could not reach a point for constructing a consistent quantum field theory. Some early investigations of a Dirac-type equation for superluminal fermion are listed in Ref. [6].

In this paper, we will start with a different approach to derive a new Dirac-type equation for superluminal neutrinos. In order to avoid introducing imaginary mass, Eq. (2) can be rewritten as

$$E = (c^2 p^2 - m_s^2 c^4)^{1/2} \quad (6)$$

where m_s is called proper mass, for instance, $m_s(\nu_e) = 1.6$ eV from Eq. (1). We follow Dirac's search [7], Hamiltonian must be first order in momentum operator $\hat{\mathbf{p}}$:

$$\hat{E} = (c\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta_s m_s c^2) \quad (7)$$

with $(\hat{E} = i\hbar\partial/\partial t, \hat{\mathbf{p}} = -i\hbar\nabla)$, where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ and β_s are 4×4 matrix, which are defined as

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}$$

and

$$\beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \quad (8)$$

where σ_i is 2×2 Pauli matrix, I is 2×2 unit matrix. Notice that β_s is a new matrix, which is different from the one in the traditional Dirac

equation. We will discuss the property of β_s in a later section.

When we take square for both sides in Eq.(7), and consider the following relations,

$$\begin{aligned}\alpha_i\alpha_j + \alpha_j\alpha_i &= 2\delta_{ij} \\ \alpha_i\beta_s + \beta_s\alpha_i &= 0 \\ \beta_s^2 &= -1\end{aligned}\tag{9}$$

the relation in Eq.(6) is reproduced. Since Eq. (6) is related to Eq. (5), this means β_s is a right choice to describe neutrinos as superluminal particles.

We now study the spin-1/2 property of neutrino as a superluminal fermion. For this purpose, we introduce a transformation,

$$\alpha \rightarrow -\alpha\tag{10}$$

Denote the wave function as

$$\Psi = \begin{pmatrix} \varphi(\mathbf{x}, t) \\ \chi(\mathbf{x}, t) \end{pmatrix}$$

with

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}\tag{11}$$

the new Dirac- type equation, Eq. (7), can be rewritten as a pair of equations:

$$\begin{aligned}i\hbar\partial\varphi/\partial t &= i\hbar\boldsymbol{\sigma} \cdot \nabla\chi + m_sc^2\chi \\ i\hbar\partial\chi/\partial t &= i\hbar\boldsymbol{\sigma} \cdot \nabla\varphi - m_sc^2\varphi\end{aligned}\tag{12}$$

Eq. (12) is an invariant under the space-time inversion transformation with $(\mathbf{x} \rightarrow -\mathbf{x}, t \rightarrow -t)$ [8,9],

$$\varphi(-\mathbf{x}, -t) \rightarrow \chi(\mathbf{x}, t), \quad \chi(-\mathbf{x}, -t) \rightarrow \varphi(\mathbf{x}, t)\tag{13}$$

From the equation (12), the continuity equation is derived:

$$\partial\rho/\partial t + \nabla \cdot \mathbf{j} = 0\tag{14}$$

and we have

$$\rho = \varphi^+ \chi + \chi^+ \varphi, \quad \mathbf{j} = -c(\varphi^+ \sigma \varphi + \chi^+ \sigma \chi) \quad (15)$$

where ρ and \mathbf{j} are probability density and current; φ^+ and χ^+ are the Hermitian adjoint of φ and χ respectively.

Considering a plane wave along x axis for a left-handed particle ($\sigma \cdot \mathbf{p} = -p$), the equations (12) yields the following solution:

$$\chi = [(cp - m_s c^2)/E]\varphi \quad (16)$$

We now consider a linear combination of φ and χ ,

$$\xi = (2)^{-1/2}(\varphi + \chi), \quad \eta = (2)^{-1/2}(\varphi - \chi) \quad (17)$$

where $\xi(\mathbf{x}, t)$ and $\eta(\mathbf{x}, t)$ are two-component spinor functions. In terms of ξ and η , Eq. (15) becomes

$$\rho = \xi^+ \xi - \eta^+ \eta, \quad \mathbf{j} = -c(\xi^+ \sigma \xi + \eta^+ \sigma \eta) \quad (18)$$

Here the normalization condition $\int \rho dx = 1$ for the wave function could be interpreted as the conservation property of helicity of particle in the motion. In terms of Eq. (17), the equation (12) can be rewritten as

$$\begin{aligned} i\hbar \partial \xi / \partial t &= i\hbar \sigma \cdot \nabla \xi - m_s c^2 \eta \\ i\hbar \partial \eta / \partial t &= -i\hbar \sigma \cdot \nabla \eta + m_s c^2 \xi \end{aligned} \quad (19)$$

In the above equations, both ξ and η are coupled via nonzero m_s .

In order to compare Eq. (19) with the well known two-component Weyl equation, we take a limit $m_s = 0$, then the first equation in Eq.(19) reduces to

$$\partial \xi / \partial t = c \sigma \cdot \nabla \xi \quad (20)$$

and the second equation in Eq. (19) vanishes because $\varphi = \chi$ when $m_s = 0$.

Eq. (20) is the two-component Weyl equation for describing neutrinos, which is related to the maximum parity violation discovered in

1956 by Lee and Yang [10,11]. They pointed out that no experiment had shown parity to be good symmetry for weak interaction. Now we see that, in terms of Eq.(19), once if neutrino has some mass, no matter how small it is, two equations should be coupled together via the mass term while still respecting the maximum parity violation.

Let us now discuss the property of matrix β_s in Eq. (8). Notice that it is not a 4×4 hermitian matrix. However, based on the above study, we now realize that the violation of hermitian property is related to the violation of parity. Though a non- hermitian Hamiltonian is not allowed for a subluminal particle, it does work for superluminal neutrinos. Besides, it still preserves the invariance of basic symmetry shown in Eq. (13).

Based on the above study, the consequences derived from the new Dirac- type equation agree with all known properties of neutrinos. Therefore, we have reached an inevitable conclusion that neutrinos must be superluminal fermions with permanent helicity.

According to special relativity [12], if there is a superluminal particle, it might travel backward in time. However, a re-interpretation rule has been introduced since the Sixties [2-3]. Another approach is to introduce a kinematic time under a non-standard form of the Lorentz transformation [13-16]. Therefore, special relativity can be extended to space-like region, and superluminal particles are allowed without causality violation.

We are grateful to Prof. S.Y. Zhu for helpful discussions. One of the authors (Chang) wishes to thank Prof. S.T. Wu, P. Richards and Dr. G. Germany for their support.

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